



# PRECISION MEASUREMENT OF AMPLIFIER NOISE TEMPERATURES BELOW 5 K

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NIST-Boulder, 8/25/05



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## I. INTRODUCTION

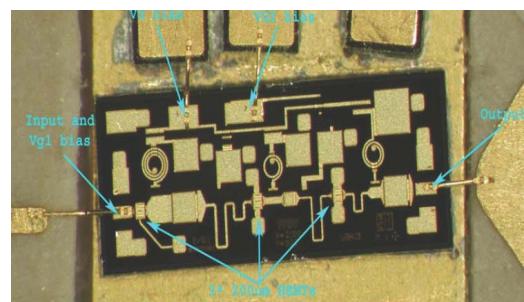
NIST  
NOISE

### Background

- Motivation
  - Cryogenic (liquid He) MMIC amplifier for IF (~ 1 – 5 GHz) section of THz receiver.
  - THz receivers used in radio astronomy, imaging (homeland security), biological imaging & diagnostics
  - Looked interesting, relatively quick, & relatively easy.
  - Proved to be more “interesting” than anticipated & therefore not so quick or easy.
  - Quickly escalated into an ATP project to build a noise measurement system (radiometer + standards) for THz.

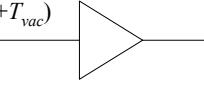
### • Amplifier

- designed & built by S. Weinreb & N. Wadefalk (JPL).
- 3 stage, InP HEMTs, 0.1  $\mu$ m gate length.
- Gain ~ 10 dB/stage, noise temp  $< \sim 10$  K from 1 – 10 GHz.
- MMIC chip  $0.75 \times 2$  mm



## Amplifier Noise Temperature

- Effective input noise temperature:

$$N_{in} = k_B(T_{in} + T_{vac})$$


$$N_{out} = GN_{in} + N_{amp} = Gk_B(T_{in} + T_{vac} + T_e)$$

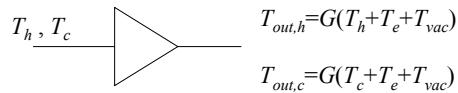
where  $N_{amp} = Gk_B T_e$

So  $T_{out} = G(T_{in} + T_e + T_{vac})$

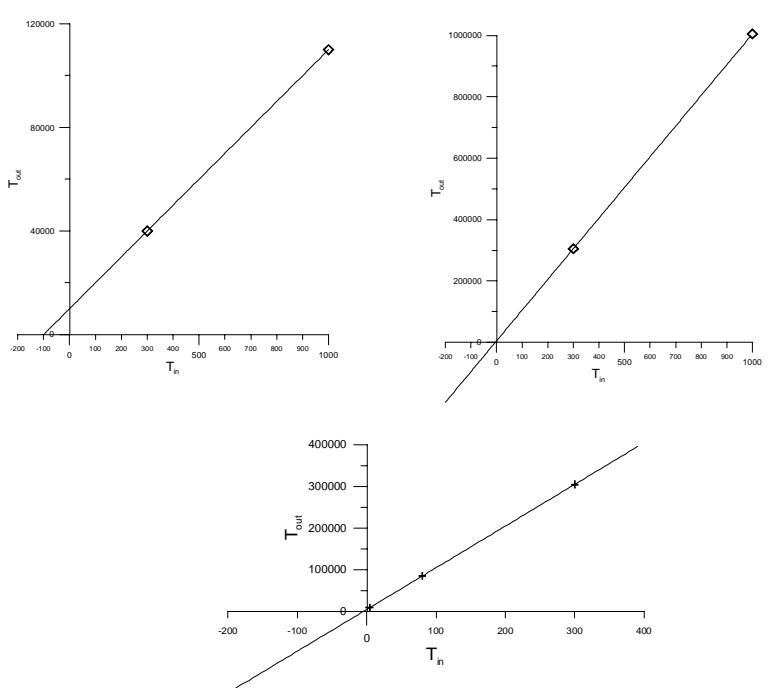
- Not-so-subtle subtleties:
  - $kT$  is *available* spectral power;  $G$  is *available* gain
  - $G$  and  $T_e$  depend on  $\Gamma_{in}$  (or  $Z_{in}$ )

- Vacuum contribution
  - $T_{vac}$  is the contribution to input noise due to vacuum fluctuations,  $T_{vac} = hf/2k_B$ .
  - Usually too small to worry about at microwave frequencies,  $T_{vac} = 0.024$  K at 1 GHz, 0.29 K at 12 GHz.
  - Our uncertainties will be as low as 0.3 K, so we need to account for it.
  - We treat it as present at the input, rather than as added by the amplifier.

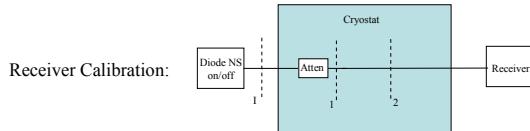
- So, have to either characterize  $T_e(\Gamma_{in})$  by measuring the noise parameters, or can measure for a given  $\Gamma_{in}$  ( $\Gamma_{in} = 0, Z_{in} = 50 \Omega$ ).
- We will do  $\Gamma_{in} = 0$  case. (Noise parameters at a cryogenic reference plane would be *very* difficult—to do well).
- “Normally” measure  $T_e \equiv T_e(\Gamma_{in} = 0)$  by hot/cold “Y-factor” method,



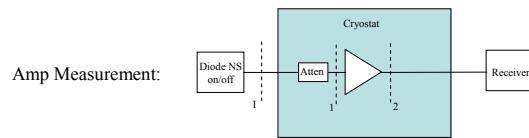
$$\text{Then } G = \frac{T_{out,h} - T_{out,c}}{T_h - T_c}, \quad T_e = \frac{T_h - YT_c}{Y-1} - T_{vac}, \quad Y = \frac{T_{out,h}}{T_{out,c}}$$



- Usual Cryogenic Amplifier Measurements  
(Cold Attenuator Method):



$$T_{1,on/off} = \alpha T_{I,on/off} + (1 - \alpha) T_{He}$$

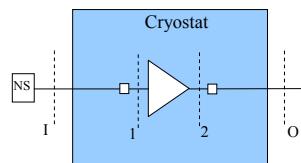


At Best:  $u_{Te} \approx 0.6 \text{ K}$  or  $0.7 \text{ K}$

Typical:  $u_{Te} > 1 \text{ K}$

## II. THEORY

- Cryogenic Complication:
  - Can measure at planes I and O (cryostat ports)
  - Want  $T_e$  and  $G$  between planes 1 and 2 (inside cryostat).



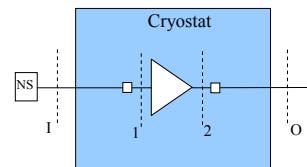
- So “just” characterize the lines, I-to-1 and 2-to-O, & correct for them.

- Basic Equations:

$$T_2 = G(T_1 + T_e + T_{vac})$$

$$T_O = \alpha_{02}T_2 + \Delta T_2$$

$$T_1 = \alpha_{1I}T_I + \Delta T_1$$



So

$$T_O = \alpha_{1I}\alpha_{02}G T_{in} + \alpha_{02}G\Delta T_1 + \alpha_{02}G(T_e + T_{vac}) + \Delta T_2$$

Want to know

Can Measure ↑

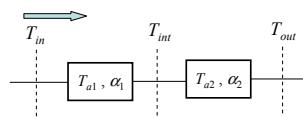
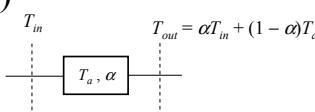
Know ↑

Must Determine ↑

Know ↑

2 different input  $T$  here will determine  $\alpha_{1I}\alpha_{02}G$

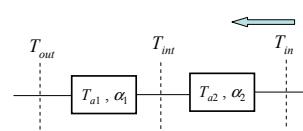
- Direction dependence of  $\Delta T_{1\text{or}2}$  (even for perfect matching)



$$T_{out} = \alpha_2 T_{int} + (1 - \alpha_2) T_{a2}$$

$$= \alpha_1 \alpha_2 T_{in} + \alpha_2 (1 - \alpha_1) T_{a1} + (1 - \alpha_2) T_{a2}$$

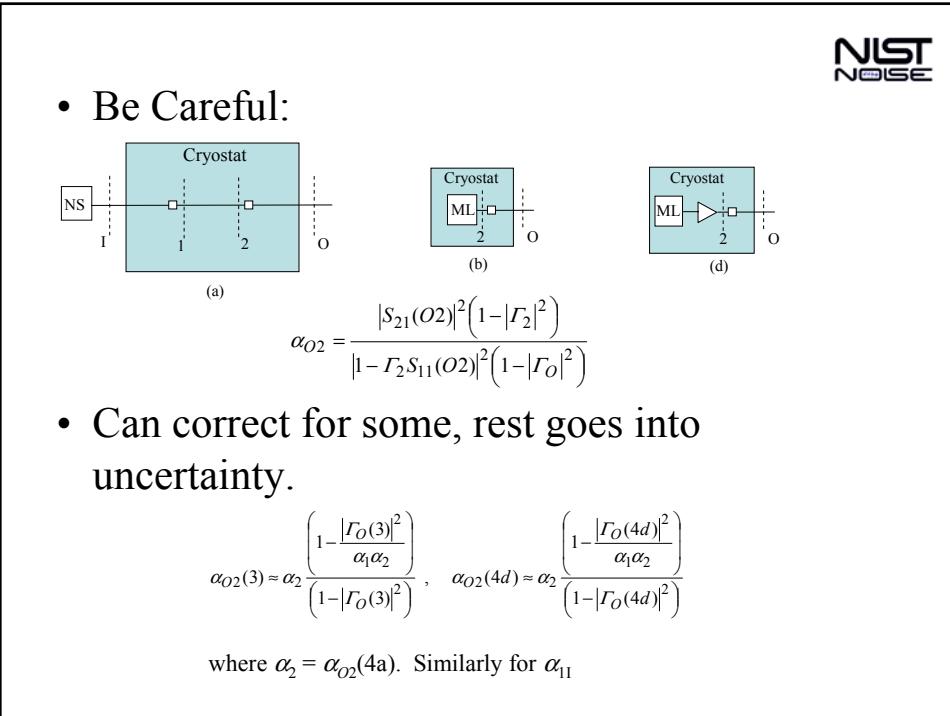
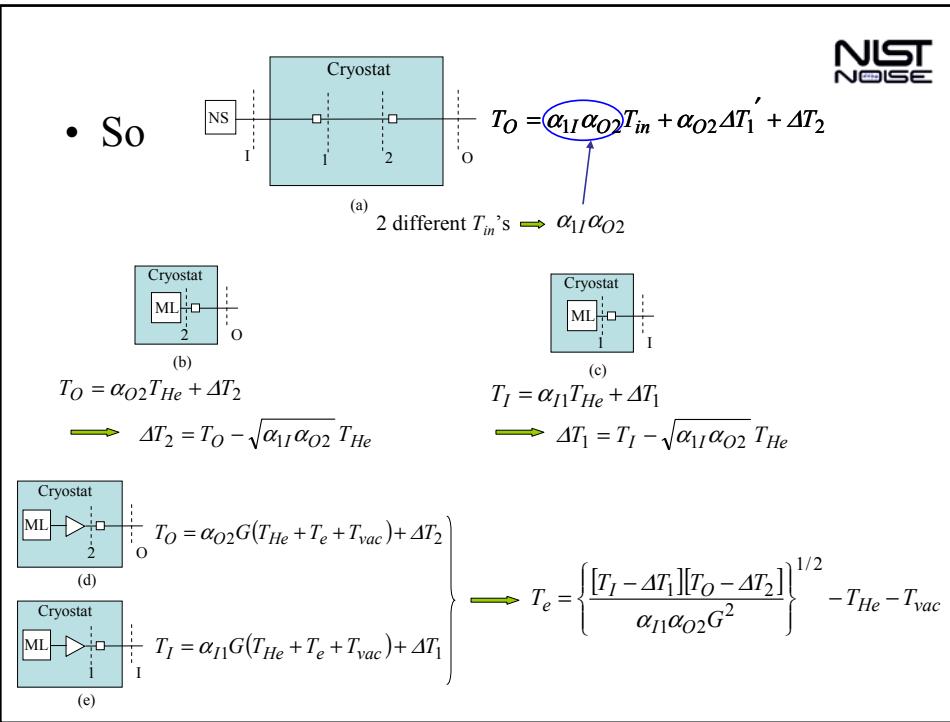
$$\Delta T = \alpha_2 (1 - \alpha_1) T_{a1} + (1 - \alpha_2) T_{a2}$$



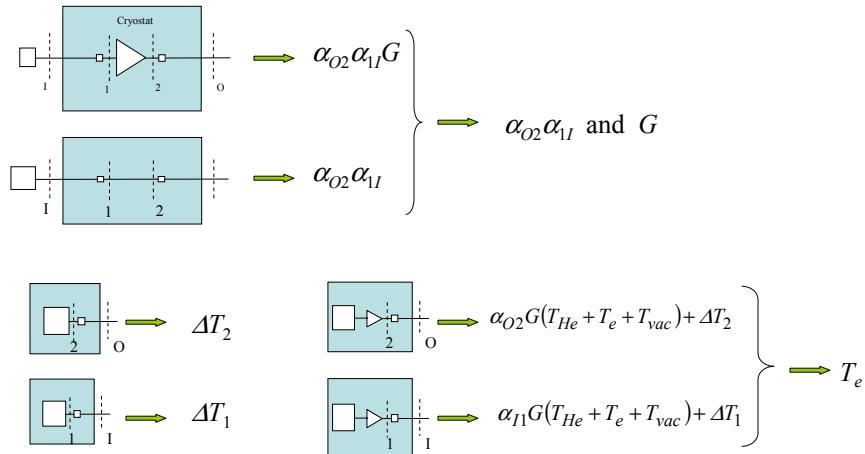
$$T_{out} = \alpha_1 \alpha_2 T_{in} + \alpha_1 (1 - \alpha_2) T_{a2} + (1 - \alpha_1) T_{a1}$$

$$\Delta T = \alpha_1 (1 - \alpha_2) T_{a2} + (1 - \alpha_1) T_{a1}$$

$\neq$



- So the general idea is:



### III. MEASUREMENTS & RESULTS

#### Setup

- Cooled to 4.1 K, biased & stabilized, then measured on VNA, then on NFRad.



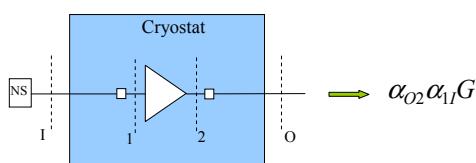
- Cryostat ports fitted with adapters (SMA to GPC-7) and with water jackets.



- Cryogenic (liquid nitrogen) source used as one of input loads for amplifier.

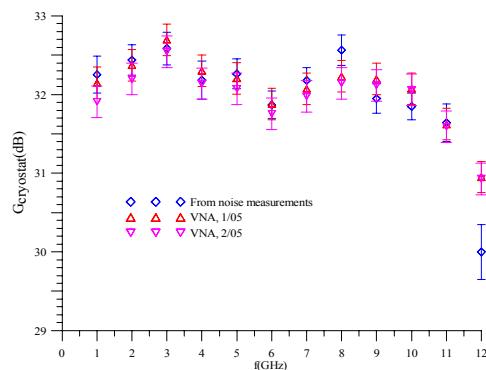


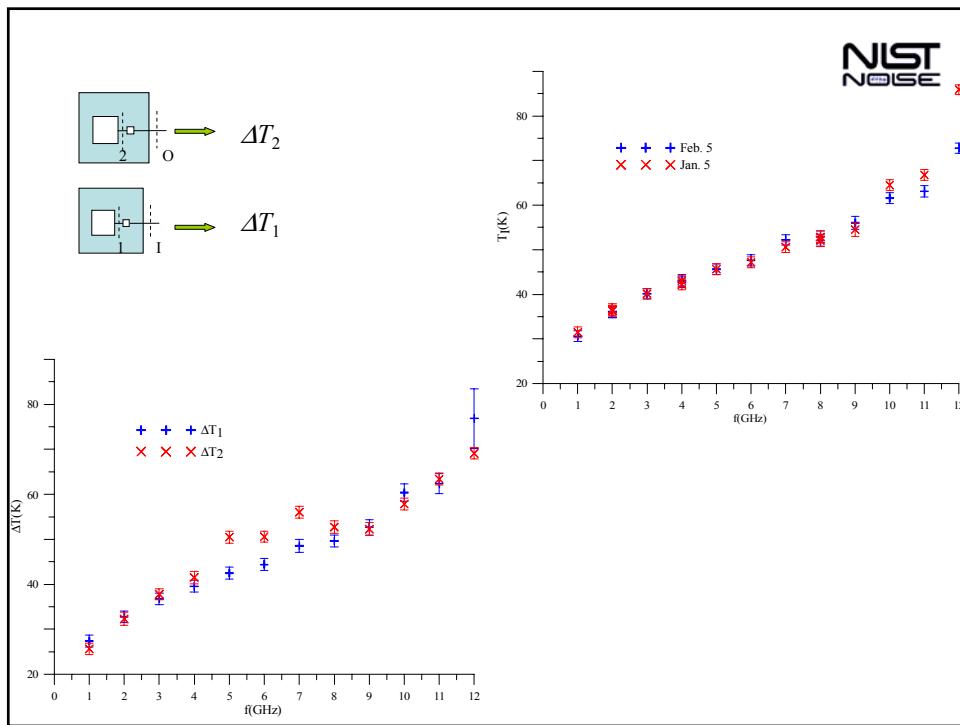
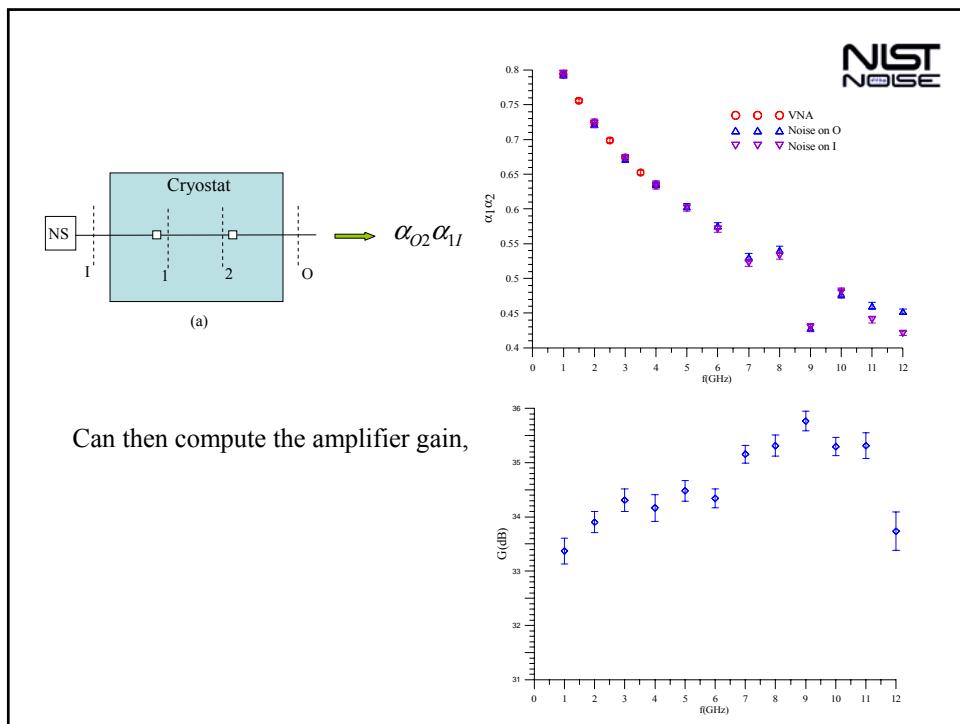
## Results

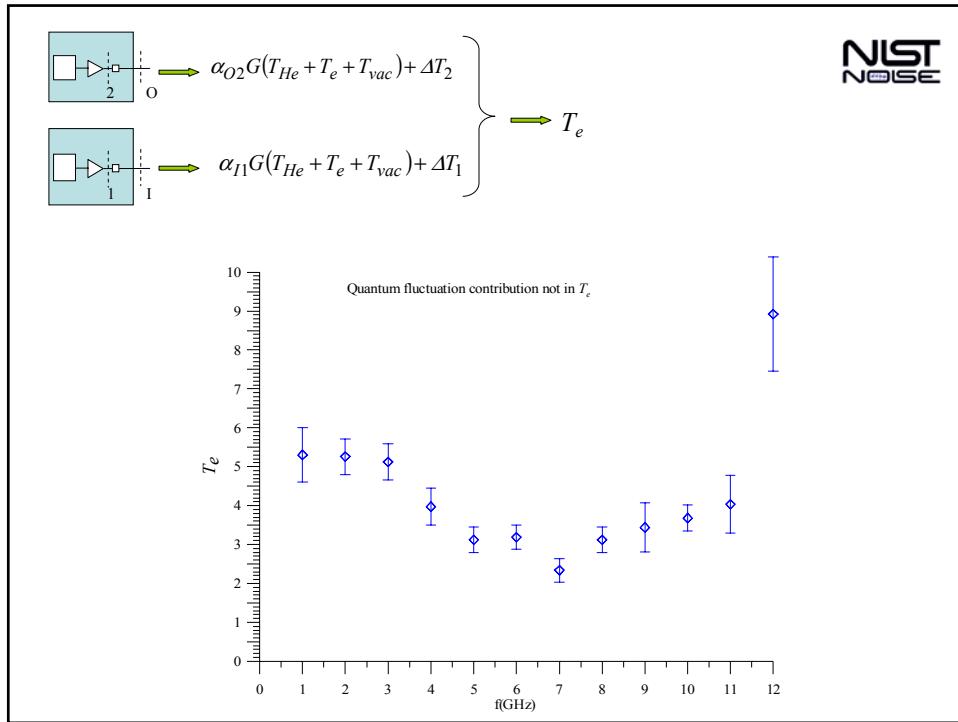


Measured with 3 different input noise sources (about 1100 K, 296 K, & 80 K). Did not use the 1100 K results because NFRad out of linear range.

Also measured with VNA for comparison (twice, for repeatability).







## IV. UNCERTAINTY ANALYSIS

- Uncertainty in  $G$ :

$$G = \frac{\alpha_{I1}\alpha_{O2}G}{\alpha_{1I}\alpha_{O2}} \quad \left\{ \begin{array}{l} \alpha_1\alpha_2 G = \frac{T_O(3,h) - T_O(3,c)}{T_h - T_c} \frac{\left(1 - |\Gamma_O(3)|^2\right)}{\left(1 - \frac{|\Gamma_O(3)|^2}{\alpha_1\alpha_2}\right)} \\ \alpha_1\alpha_2 = \frac{T_O(4a,h) - T_O(4a,c)}{T_h - T_c} \end{array} \right.$$

$$\frac{u(G)}{G} = \sqrt{\frac{u^2(\alpha_1\alpha_2 G)}{(\alpha_1\alpha_2 G)^2} + \frac{u^2(\alpha_1\alpha_2)}{(\alpha_1\alpha_2)^2} + \frac{u_{ML}^2(G)}{G^2}}$$

due to imperfect matches  
 $\approx 0.6\% \text{ to } 6\%$ , depending on frequency

$$\frac{u(\alpha_1 \alpha_2 G)}{\alpha_1 \alpha_2 G} = \sqrt{\frac{u^2(\Delta T_O)}{\Delta T_O^2} + \frac{u^2(\Delta T_{in})}{\Delta T_{in}^2} + \frac{u_{ML}^2(\alpha_1 \alpha_2 G)}{(\alpha_1 \alpha_2 G)^2}}$$

normally  $\approx 0.6\%$   
but repeatability  $\Rightarrow \approx 3\% \text{ or } 4\%$

Therefore,  $\frac{u(\alpha_1\alpha_2G)}{\alpha_1\alpha_2G} \approx 3\% \text{ to } 8\%, \text{ depending on frequency}$

Similar analysis  $\Rightarrow \frac{u(\alpha_1\alpha_2)}{\alpha_1\alpha_2} \approx 0.5\%$

$$\text{So } \frac{u(G)}{G} = \sqrt{\frac{u^2(\alpha_1\alpha_2 G)}{(\alpha_1\alpha_2 G)^2} + \frac{u^2(\alpha_1\alpha_2)}{(\alpha_1\alpha_2)^2} + \frac{u_{ML}^2(G)}{G^2}} \approx 3\% \text{ to } 8\%, \text{ depending on frequency}$$

- Uncertainty in  $T_e$

$$T_e = \left\{ \frac{[T_I(4d) - \Delta T_1][T_O(4e) - \Delta T_2]}{\alpha_{I1}(4e)\alpha_{O2}(4d)G^2} \right\}^{1/2} - T_{He} - T_{vac}$$

In evaluation, use  $\alpha_{I1}\alpha_{O2}G^2 = \frac{(\alpha_{I1}\alpha_{O2}G)^2}{\alpha_{I1}\alpha_{O2}}$

Making reasonable estimates for noise parameters of amplifier, based on  $T_e$ , we get

$$\frac{u_{ML}(T_e)}{T_e} \approx \sqrt{4|\Gamma_{ML}|^2 + 2|\Gamma_{ML}S_{11}|^2} \approx <1 \% \text{ to } 8.5 \%$$

- Combining it all, we get

$f$ (GHz)	$G$ (dB)	$T_e$ (K)
1	$33.4 \pm 0.3$	$5.3 \pm 0.7$
2	$33.9 \pm 0.2$	$5.3 \pm 0.5$
3	$34.3 \pm 0.2$	$5.1 \pm 0.5$
4	$34.2 \pm 0.3$	$4.0 \pm 0.5$
5	$34.5 \pm 0.2$	$3.1 \pm 0.3$
6	$34.3 \pm 0.2$	$3.2 \pm 0.3$
7	$35.1 \pm 0.2$	$2.3 \pm 0.3$
8	$35.3 \pm 0.2$	$3.1 \pm 0.3$
9	$35.8 \pm 0.3$	$3.4 \pm 0.6$
10	$35.3 \pm 0.2$	$3.7 \pm 0.3$
11	$35.3 \pm 0.4$	$4.0 \pm 0.7$
12	$33.7 \pm 0.5$	$8.9 \pm 1.5$

## V. CONCLUSIONS

- We have measured amplifier noise temperatures as low as 2.3 K with a standard uncertainty of  $\pm 0.3$ K  
(note: noise figure =  $0.034 \text{ dB} \pm 0.004 \text{ dB}$ )
- Factor of 2 better uncertainties than previous best.
- Amplifier has excellent performance over a wide frequency range,  $T_e < 5.5$  K from 1 – 11 GHz.

- Improvements & plans:
  - Better connectors, cables, matched loads(?)
  - 3 input noise sources (?)
  - Cryocooler
  - Characterize, automate
  - Measurements on mixers (?).